

On the interaction between indirect cost allocations and the firm's objectives

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Abstract:

The import of cost allocation procedures are through their ex ante impact on decision making. Hence, it is important that the allocation issue be placed squarely within the context of those firm's objectives which gave rise to the need for the specific allocation. To that effect, this paper focuses the debate on the identification of the indirect cost allocation method that is best suited to the specific reasons for requiring the cost information. First, it is shown that all existing allocation schemes (i) may be expressed in a common equation, flexible enough to be adapted to whatever decision-making purpose the firm desires; and (ii) fulfil the individual rationality conditions of game theory. Then, in light of the controversy as to whether the US Defense Department indirectly subsidizes the commercial side of its suppliers' operations, necessary and sufficient conditions are provided for allocations which do and do not subsidize. Non-subsidized allocations are shown to belong to the core. Subsidized allocations occur when the players (divisions') rational objectives are superseded by higher priority coordinating objectives of non-players (the firm).

Keywords: Cost allocation; Accounting; Core; Inventory; Optimization

Article:

1. Introduction

This paper deals with the allocation of joint costs in cost-plus pricing situations. Its initial motivation lies on the question as to whether the US Defense Department indirectly subsidizes the commercial side of its suppliers' operations. Our points of departure are the 1987 inquiry of the US Defense Contract Audit Agency (e.g. [10]), the October 1992 issue of *Accounting Review* on a Forum on Accounting for Defense Contracts [3,11,15,17,26] and the recent articles of Gerchak and Gupta [7] and of Robinson [16], which have revived the controversy surrounding the indirect cost allocation issue.

Rogerson [17] argues that the Department's practice of allocating overhead as a function of direct labour costs encourages firms to shift government procurement to in-house production and, if necessary, commercial output towards subcontracting. Similarly, Thomas and Tung [26] argue that contractors tend to overfund those pension plans associated with defense-related personnel and underfund those of workers engaged in commercial businesses. To generalize the subsidization issue, Gerchak and Gupta [7] use the continuous review order quantity reorder point inventory model with backordering (e.g. [23]), to show that allocation schemes based on volume, safety stocks or incremental contribution to total costs may not be fair, in the sense that at least one customer pays more than under the stand-alone cost policy. Instead, an allocation in proportion to the separable or non-centralized costs seems to be preferable for all customers. Robinson [16] recasts the problem within a game-theoretical framework and shows that the recommended allocation of Gerchak and Gupta [7] is not in the core, whereas those based on the Shapley value [22], Moriarity's [13] and Louderback's [12] are.

A common thread of all these studies is their emphasis on the question of *how to allocate* at the expense of *what to allocate*. As a result, the discussion has centred on the development of a variety of allocation schemes, where

the differences lie (i) on the properties of the criteria to be used, such as whether the particular allocation scheme belongs to the core; and/or (ii) on the objectives of the allocation method, be them size, variability and the like. This has lead to three types of indirect cost allocation schemes proposed in the literature. One consists of game-theory based schemes, such as the Shapley [22] value and its variants (e.g. [8,9,18]). Another includes normative measures like those of Moriarity [13], Louderback [12], Gangolly [6] and Balachandran and Ramakrishnan [1]. The third one uses activity or ability-to-bear measures, such as size, variability, sales, plant assets, net realizable value and the like, as the basis for allocation (e.g. [2,5,19-21]).

Whereas this line of reasoning continues to be a fruitful avenue of research, it does not address directly another fundamental question, namely what is being allocated. Addressing such question is the primary purpose of this study. The basic premise is that, since cost allocation procedures have direct impact on decision making within the firm, it is not possible to view cost allocations as separate from the incentives for divisions to make decisions within the firm. In fact, the value of a method will differ depending on whether the cost information is used for make or buy decisions, pricing, incentive compensation and the like. Nevertheless, it is possible to design a Generalized Benefit-Allocation Scheme (GBAS), where the main concern is how to translate such variety of incentives into 'savings' to be shared among the divisions of the firm and where the criteria for the sharing of those 'savings' are left entirely at the discretion of the firm's decision-making objectives for carrying out the allocation.

The formulation of the GBAS represents the first main contribution of the paper. It results in a generalized 'how to' method, where the incentives are all converted into 'savings' from centralization in a similar fashion, even if the attributes of the 'best' allocation scheme can only be specified within the context of the decision making situation. It also indicates what to allocate, namely the benefits of centralization. The GBAS is described in the next section. It is shown (i) to be fair and valid as long as the cost associated with the grand coalition of all centres does not exceed the sum of the stand-alone costs; and (ii) to encompass the existing allocation models.

The second contribution consists of the identification and comparison of two approaches to allocate the benefits of centralization. In light of the Defense Department example, they differ on the firm's objective for the allocation. The first (Section 3) assumes that no cross-subsidization of divisions is to occur. Hence, each division represents an independent and rational player, seeking as the only objective to maximize its share of the benefits of centralization. Allocations satisfying this assumption are shown to belong to the core and necessary and sufficient conditions for their existence are provided. The second (Section 4), which corresponds to the Defense Department example, considers the same players now constrained in the pursuit of its optimizing objective by the overall profitability of a non-player, namely the firm. The subsequent analysis brings into question the usefulness of core allocations, when player's rational objectives are superseded by higher-priority coordinating objectives of non-players. To illustrate this point, a simple game is created (Section 5) and optimal solutions consistent with the subsidizing behaviour alluded to earlier are provided. Finally, a Conclusions section completes the paper.

2. A family of fair allocation schemes

This well-known property, related to the benefits of centralization, is a necessary condition for the acceptability of any indirect cost allocation scheme.

Property 1 (Centralization pays). *Consider N cost centres, each with stand-alone costs of C_i and let C_T be the cost associated with the centralized system. Then, the total costs under a single centralized centre does not exceed the sum of all the individual stand-alone costs, i.e.*

$$C_T \leq \sum_{i=1}^N C_i \quad (1)$$

One of the most common applications of (1) lies in the area of risk pooling, as in random-demand inventory systems (e.g. [27]). Another deals with the elimination of redundant transfers in the centralizing of cash management of a multinational business (e.g. [4]).

In this section, we consider the proposition that any of allocation schemes alluded to above may result in allocations smaller than the stand-alone costs, if the question is phrased in terms of what to allocate rather than how to allocate. For this purpose, the following definitions are needed.

Definition 1 (*Allocation scheme*). An allocation scheme represents the portion, X_i , $i = 1, \dots, N$, allocated to each player i of the costs, C_T , incurred by the grand coalition of all N players.

Definition 2 (*Fair allocation schemes*). An allocation scheme of the joint costs, C_T , is said to be fair if the following conditions hold:

- (a) It allocates to each centre i an amount, X_i , not exceeding its stand alone cost, C_i i.e.

$$X_i \leq C_i \quad (2)$$

- (b) The total cost, C_T , is fully allocated, i.e.

$$\sum_{i=1}^N X_i = C_T$$

Allocations which satisfy (2) and (3) are also said to satisfy the individual rationality criterion of game theory. Hence, (2) and (3) characterize the imputations of the game. These allocations also fulfil Thomas' [24,25] additivity and unambiguity allocation criteria, since the entire joint costs must be allocated and in a unique way.

On the basis of these results, the following property characterizes allocation sets which are consistent with (2) and (3).

Property 2 (A family of fair allocation schemes). *All allocations of the form*

$$\begin{aligned} X_i &= C_i - \delta \alpha_i, i = 1, \dots, N, \\ \sum_{i=1}^N \alpha_i &= 1, \\ 0 &\leq \alpha_i \leq 1, \\ \delta &= \sum_{i=1}^N C_i - C_T, \end{aligned} \quad (4)$$

with α_i defined as the allocation rate, are fair, in the sense of Definition 2.

The import of this family of allocations is manifested in at least four significant ways. First, the functional form has an intuitively appealing economic interpretation. The joint cost allocated to centre i , X_i , equals the stand-alone cost, C_i , minus a fraction, α_i , of the coordination gain or stand-alone penalty. Such family of allocations clearly determines what is to be allocated to i , namely a fraction, α_i , of the savings, arising from the formation of the grand coalition. Second, since, by Property 1, this fraction is always non-negative, any allocation scheme defined by (4) is fair in the sense of (2) and (3). Third, as Appendix A shows, this includes the normative and game-theory schemes discussed above, i.e. the Moriarity, Gangolly, Shapley, Louderback, and Balachandran

and Ramakrishnan allocation schemes. Hence, all of these allocation schemes may be placed in the same format, regardless of the purpose of the allocation.

Fourth and more importantly, since a_i has no units of measurement, it does not necessarily have to represent a cost ratio. In fact, the allocation proportion, a_i , may be computed on the basis of any ability-to-bear measure, while still keeping the corresponding GBAS fair. The main implication of this result is that it places the choice of measures squarely in the hands of the firm. The restriction that a_i is computed on an indirect-cost basis is lifted. The firm may tailor the allocation scheme to suit the objectives of the particular decision making situation requiring the allocation. This allows management to reward centres with, say, higher sales or lower lead time demand, rather than higher costs. It may also help to alleviate the allocation problem in the Defense Department situation, if a switch in allocation basis is made from direct labour costs to other value measure which does not reward higher costs.

3. Fair and equitable allocations

If the question of whether a centre is subsidizing another is relevant, as is in the US Defense Department example mentioned earlier, then it should be first observed that not all solutions satisfying Property 2 yield non-subsidized allocations. For example, Robinson [16] shows numerically that Gerchak and Gupta's [7] preferred allocation scheme penalizes the largest centre. Hence, additional criteria are needed to ensure that the resulting scheme yields Pareto-optimal allocations. To that effect, the following definition is needed.

Definition 3 (*Fair and equitable allocation schemes*). Let C_S be the cost for sub-coalition S , composed of any combination of the members of N , i.e. $S \subseteq N$, and C_{-S} , its counterpart for a coalition composed of all members in N but not in S . Then, a fair allocation is said to be *equitable* if it satisfies the group rationality criterion of game theory, i.e. if, in addition to (2) and (3), the following condition holds:

$$\begin{aligned} \sum_{i \in S} X_i &\leq C_S \text{ for all } S \subseteq N \\ \Leftrightarrow \sum_{i \notin S} X_i &\leq C_{-S} \text{ for all } S \subseteq N. \end{aligned}$$

Hence, any scheme which does not include all centres yields nonequitable and hence suboptimal allocations. Thus the set of all equitable imputations form the core of the game. Its existence is not in doubt, since Balachandran and Ramakrishnan [1] prove that the L/BR allocation is always in the core.

The next result identifies the necessary and sufficient conditions for an allocation scheme, X_i , $i = 1, \dots, N$, to belong to the core.

Lemma 1 (Core allocations). *Let I_S be the incremental cost incurred if S joins all others ($-S$) in a grand coalition. Then, an allocation scheme, X_i , $i = 1, \dots, N$, is in the core iff*

$$I_S = C_T - C_{-S} \leq \sum_{i \in S} X_i \text{ for all } S \subseteq N. \quad (6)$$

Proof. By contradiction. Suppose that the allocation set belongs to the core, i.e. it satisfies Definition 3. Assume further that, contrary to (6),

$$I_S = C_T - C_{-S} \leq \sum_{i \in S} X_i \text{ for some } S \subseteq N. \quad (7)$$

Then, combining (7) and (3), one obtains

$$C_T - C_{-S} \geq C_T - \sum_{i \notin S} X_i$$

and

$$\sum_{i \notin S} X_i \geq C_{-S} \quad (8)$$

which violates the core condition in (5); hence, the first contradiction.

Similarly, suppose that (6) holds and consider, contrary to (5), that

$$\sum_{i \in S} X_i \text{ for some } S \subseteq N \quad (9)$$

Then, combining (9) and (3), one obtains

$$C_T \sum_{i \notin S} X_i \geq C_S$$

and

$$\sum_{i \in S} X_i \leq C_T - C_S \quad (10)$$

which violates the condition in (6); hence, the second contradiction.

From Definition 3 and Lemma 1, it follows that all equitable allocation schemes, X_i , $i = 1$, must satisfy

$$C_T - C_{-S} \leq \sum_{i \in S} X_i \leq C_S \text{ for all } S \subseteq N.$$

Expression (11) places in the core all schemes which allocate to each coalition, $S \subseteq N$, (i) at least its marginal contribution, $C_T - C_{-S}$, to the grand coalition's cost; but (ii) no more than the cost incurred by S on its own. The first inequality is needed to avoid S being subsidized by the other members, whereas the second puts an upper limit on the cost to be allocated to S , as an incentive for S to join those in $(-S)$. Additional insights into the nature of the core allocations are provided by the next lemma, obtained when (4) and (11) are combined to produce another set of necessary and sufficient conditions for a set of allocations to be in the core.

Lemma 2 (An alternate characterization of core allocations). *An allocation pattern is in the core iff*

$$\sum_{i \in S} C_i - C_S \leq \delta \sum_{i \in S} \alpha_i \leq \sum_{i \in S} C_i + C_{-S} - C_T. \quad (12)$$

Expression (12) conveys a very intuitively appealing economic interpretation to equitable allocations. Since α_i represents the proportion of the coordination gain, δ , allocated to centre i , it follows that the middle term yields the portion of the consolidation gains allocated to those in S . As an incentive to join the grand coalition, this amount must exceed those obtained by the members of S forming a coalition among themselves only. This

justifies the direction of the first inequality. As for the second and to avoid subsidizing, S's share of the consolidation gains cannot exceed its contribution towards the cost reduction for the grand coalition.

4. Are equitable allocations necessary?

The desirability of core allocations lies on the premise that players are rational and self-interested. Thus, Pareto-optimal solutions yield the least undesirable allocations. However, the Defense Department example brings an additional element into the equation, namely that the firm itself has objectives of its own which may supersede those of the individual divisions. As a result, it may be perfectly rational behaviour for the firm to allocate to a given centre (e.g. the one with the government contract) more than its share of the costs, if market conditions allow it. A case in point, even if extreme, is a firm with a monopoly product to which a larger than equitable share of the indirect costs is allocated, to help render other products more competitive. In more realistic cases and generalizing the 'ability-to-bear' argument, allocations in inverse proportion to demand elasticities may yield 'better' allocations for the firm ('better', in terms of the firm's overall objective), even if not necessarily for all the centres themselves, than their Pareto-optimal counterparts, since the former are more representative of market conditions.

Another example may be that of the government contract alluded to earlier. At its simplest, the problem may be posed as a comparison between one (or more than one) player(s), g , representing the government contract(s) and all the others, $(-g)$, which represent the commercial side of the firm. Then, a potential solution to the allocation issue may be based on a game where the only coalitions allowed are the commercial players on the one hand and the government-contract side, assumed for simplicity to be a singleton. The following result characterizes such a game.

Lemma 3 (A special type of game). *Consider a game characterized by the following cost structure:*

$$C_S = \begin{cases} C_T & \text{if } S = N, \\ C_g & \text{if } S = g, \\ C_{-g} & \text{if } S = (-g), \\ \infty, & \text{otherwise.} \end{cases} \quad (13)$$

Then, an allocation belongs to the core iff

$$\begin{aligned} \delta(1 - \alpha_i) &\geq \phi_i \text{ for } i = 1, \dots, N, \\ \delta &= \sum_{s=1}^N C_s - C_T, \\ \phi_i &= \sum_{s \neq i} C_s - C_{-i} \end{aligned} \quad (14)$$

Expression (14), obtained from (11) and (13), also conveys a very intuitively appealing economic interpretation. For our purposes, $i = g$ is the only singleton of interest. Then, since $(1 - \alpha_g)$ yields the fraction available for sharing among all commercial divisions, $\delta(1 - \alpha_g)$ represents the cost savings available to be shared by all non-government divisions, if the grand coalition is formed. On the other hand, if g is left out on its own, the cost savings available to the singleton, ϕ_g , reflect the difference between the sum of the stand-alone costs and those from a coalition of all non-government divisions. Then, for a given allocation to be in the core, the grand coalition gains must exceed those obtained by a coalition formed by all divisions except g .

In more general terms, one may envision a firm with N divisions, each with different market power or at the very least with different 'ability to bear' capabilities, which determine the share of indirect costs allocated, on the basis of some objective on the part of the firm. The end result may not be in the core and it may not even belong to those characterized by Definition 2. Such problems may be formulated and solved by using standard

optimization techniques. For illustrative purposes, the following property considers a firm's profit-maximizing allocation problem and optimal allocation schemes for three special cases.

Property 3 (A profit-maximizing allocation problem). (a) *Let $f_i(X_i)$ be the profit earned by centre i , $i = 1, \dots, N$, before its share of the indirect costs is allocated. Then, the firm's allocation problem consists of*

$$\text{Max} \sum_{i=1}^N f_i(X_i) - C_T \quad (15)$$

s. t.

$$\sum_{i=1}^N X_i = C_T,$$

$$X_i \geq 0 \text{ for all } i = 1, \dots, N.$$

(b) *If $f_i(X_i)$ is a convex function of X_i , $i = 1, \dots, N$, then there exists an optimal allocation strategy in which all costs are allocated to only one centre, i.e.*

$$X_i^* = \begin{cases} C_T & \text{for one } i \in N \\ 0, & \text{which maximizes (15) otherwise} \end{cases} \quad (16)$$

(c) *If $f_i(X_i)$ is a concave function of X_i , $i = 1, \dots, N$, the optimal allocation pattern may be approximated by the solution to the linear program resulting from replacing in (15) all $f_i(X_i)$'s with piece-wise linear functions.*

(d) *If all $f_i(X_i)$'s are identical concave functions, then the optimal solution to (15) is to allocate an equal amount to all centres, i.e.*

$$X_i^* = C_T/N \text{ for all } i = 1, \dots, N. \quad (17)$$

Property 3a provides the mathematical formulation for a specific allocation problem, where the objective is the maximization of profits earned by all centres. Profits are defined as a function of the allocation pattern, given by the X_i 's. The constraints are the individual rationality conditions of Definition 2. Optimal solutions for special forms of the profit functions are presented in Properties 3b-3d. More specifically, Property 3b presents an optimal solution to the problem of maximizing a convex function, subject to a linear constraint. For such problems, there exists an optimal solution, given by (16), at the extreme point which yields the maximum profit. Similarly, Properties 3c and 3d present two well known cases of maximizing a concave function, subject to linear constraints. Property 3c plays the double role of giving an approximate solution for the general concave case and of serving as a necessary condition for Property 3d, whereas Property 3d provides an explicit optimal solution to the special case of identical concave functions. The validity of Property 3c is justified in Chapter 1 of [14] and that of Property 3d, in Appendix B.

Property 3b highlights some of the problems of the 'cost-plus' type of arrangements, common in many of the government contracts discussed earlier. It gives the firm the incentive to shift as much of the indirect cost burden as possible to the government side. In fact, it can be readily shown that such a result holds for any full-cost pricing situation. Furthermore, for (16) to be fair, it is required that C_T does not exceed the stand-alone cost of the centre which absorbs the entire cost. Such a situation, even if somewhat unusual, may be valid in cases of, say, a player (the government?) with very high stand-alone cost structure, at least as compared to that of the other players (on the commercial side?).

Property 3d relates to the question of what-to-allocate, either cost or benefit. Specifically, the equal cost allocation solution of (17) can be shown to be fair, only if there exists at least one player with a stand-alone cost, C_i , higher than the average cost per player, C_T/N , of the grand coalition, i.e. unless all players are of similar strength. This is in contrast to the equal benefit allocation solution of $a_i = 1/N$, which may be shown to be always fair, but not necessarily equitable.

5. Conclusions

Based on the Defense Department example alluded to earlier, this paper evaluates the question of whether it is rational behaviour for a firm to shift indirect costs from one division to another at the expense of the specific profitability objectives of the affected divisions. To answer this question it is worth noting that the import of cost allocation procedures are through their ex ante impact on decision making. As a result, the allocation issue should be placed squarely within the context of those firm's objectives which gave rise to the need for the specific allocation. As shown in this paper for the Defense Department example, it is perfectly rational for a firm to shift indirect costs to particular divisions, if such undertaking better meets its overall objectives.

To that effect, this paper shifts the focus of the debate on the identification of the 'best' indirect cost allocation method to the purpose of the allocation itself. Any allocation scheme must be evaluated in context. The costs and benefits of an allocation scheme depend on how the cost information is used. For example, the Shapley allocation of the cost of an internally generated good or service keeps divisional managers from purchasing that good or service outside the company, on their own, at a higher cost to the company as a whole. Here, the cost information is used in the make or buy decision. In the Defense case, the cost information is used to set prices and generate profits when the firm is engaged in both government and commercial dealings.

This shift has resulted in the identification of an entire family of allocations, which meet the desirable individuality conditions of game theory and are flexible enough to be adapted to whatever, decision-making purpose thy firm has in mind for this cost information. From these, core allocations have also been derived, for those cases in which the firm chooses not to subsidize any division at the expense of the others. In the process, questions have been raised as to the appropriateness of game-theoretic solutions to the indirect cost allocation problem, if, as in the Defense Department example, subsidization meets the objectives of the firm. As a result, specific solutions not necessarily in the core have been shown to be desirable for the firm, even if not necessarily for the players themselves, under various realistic conditions. An important implication of this research is to question the usefulness of searching for equitable allocations in cases whereby 'coordinating' objectives have priority over and may conflict with the players' own priorities.

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Appendix A. Comparing GBAS to the normative allocation schemes

In this Appendix, the GBAS is compared to the Moriarity, Gangolly, Shapley, Louderback, and Balachandran and Ramakrishnan allocation schemes. For the GBAS, the last two schemes may be shown to be equivalent (heretofore denoted by L/BR). To that effect, Table 1 provides expressions for the allocation, X_i , and the allocation proportion, a_i , to centre $i = 1, \dots, N$, associated with each scheme. Table 2 provides an illustrative computational example, using the model and the following data from Gerchak and Gupta [7] and Robinson [16].

Annual costs:

$$\begin{bmatrix} C_1 & C_2 & C_3 \\ C_{12} & C_{13} & C_{23} \\ C_{123} \end{bmatrix} = \begin{bmatrix} 4519.91 & 901.29 & 616.91 \\ 4795.84 & 4681.68 & 1123.26 \\ 4894.85 \end{bmatrix}. \quad (\text{A.1})$$

Note that the term I_i in Table 1 denotes the incremental cost incurred if centre i joins the grand coalition. Also, observe, from the X_i 's of Table 1 and the (X_i/C_i) 's of Table 2, that Moriarity's scheme results in all centres being allocated a constant proportion to their stand-alone costs. On the other hand, L/BR's depends upon each centre's contribution to the total costs. In addition, the expressions and the values for the allocation fraction clearly delineate the special characteristics of the two allocation methods. Moriarity allocates the cooperation gain in direct proportion to the stand-alone costs. L/BR's weights are based on the stand-alone costs net of those arising from centre i joining the coalition. Furthermore, by equating their respective expressions for a_i , it can be readily shown that the two methods yield the same allocation, when centre i 's contribution rate to the stand-alone costs equal its counterpart to the incremental costs, i.e. when

$$C_i / \left(\sum_{S=1}^N C_S \right) = I_i / \left(\sum_{S=1}^N I_S \right). \quad (\text{A.2})$$

Moriarity and L/BR consider costs and benefits of a centre either standing on its own or as a member of the grand coalition. In addition, the other two schemes also take into consideration each centre's contribution to the various possible subcoalitions. Gangolly's scheme consists of two steps. First, for each subcoalition $R \subseteq N$, the savings of forming R is allocated to its members in proportion to their stand alone costs. Then, the total allocation for any given member i represents the sum of the savings allocated to all subcoalitions of which i is a member. On the other hand, Shapley allocates to each centre i an amount equal to the expected marginal savings incurred when i joins the different subcoalitions. For the example of Table 2, the Shapley's allocation proportion suggests almost equiprobable allocation of the grand coalition's savings. In addition, an examination of the appropriate allocation fractions from Table 1 suggest that Gangolly's scheme is equivalent to Moriarity's and L/BR's when (A.2) and the following expression hold true:

$$\delta = (1 - \delta_{Gi}) \sum_{s=1}^N C_s. \quad (\text{A.3})$$

Expression (A.3) has also a meaningful economic interpretation. Observe from Table 1 that, in the equation for δ_{Gi} , the value corresponding to the $R = i$ singleton is always equal to one. Hence, the term in parentheses of (A.3) represents the sum of the proportions of the cost savings incurred when i joins all potential subcoalitions. Then, for all three schemes to yield the same allocations, that cumulative fraction of the total stand-alone costs must equal δ , the benefits obtained if, rather than all acting on their own, all centres were to form a grand coalition.

Table 1
Existing allocation schemes

Allocation scheme	Allocation, X_i	Allocation fraction, α_i
Moriarity	$C_i C_T / \sum_{s=1}^N C_s$	$C_i / \sum_{s=1}^N C_s$
L/BR	$I_i + (C_i - I_i)(C_T - \sum_{s=1}^N I_s) / \sum_{s=1}^N (C_s - I_s)$	$C_i I_i / \sum_{s=1}^N (C_s I_s)$
Gangolly ^a	$C_i \delta_{Gi}$	$C_i(1 - \delta_{Gi}) / \delta$
Shapley ^b	$(C_i - \delta_{SHi}) / N$	$[C_i(N - 1) - \delta_{SHi}] / (N\delta)$

^a

$$\delta_{Gi} = \sum_{i \in R \subseteq N} \left[\sum_{S \subseteq R} (-1)^{(r-s)} C_S \right] / \left[\sum_{j \in R} C_j \right].$$

^b

$$\delta_{SHi} = \sum_{i \notin S \subseteq N} (C_{S \cup i} - C_S) / \binom{N-1}{s}.$$

Table 2
Comparing various allocation policies

Scheme		Centre 1	Centre 2	Centre 3
Moriarity	X_i	3 662.88	730.39	499.94
	X_i / C_i	81.04	81.04	81.04
	α_i	74.86	14.92	10.22
L/BR	X_i	4081.67	498.22	313.31
	X_i / C_i	90.30	55.28	50.79
	α_i	38.28	35.21	26.52
Gangolly	X_i	3 846.44	612.88	435.54
	X_i / C_i	85.10	68.00	70.60
	α_i	58.91	25.23	15.86
Shapley	X_i	4089.84	501.32	302.05
	X_i / C_i	90.48	55.62	48.96
	α_i	37.57	34.94	27.49
Annual demand	D_i	80 000.00	2 000.00	1 000.00
	α_i	96.39	2.41	1.20
	X_i	3417.92	873.74	603.19

In summary, the above comments help placing the four allocation schemes into proper perspective. A distinction is made between the allocation of costs (Moriarity) and the distribution of coalition-formation savings (L/BR, Gangolly, Shapley). In addition, the savings may be total, i.e. resulting from a comparison between the grand coalition and the singletons (L/BR), or may also include subcoalition savings (Gangolly and Shapley). In any case, the allocation basis is cost proportional. However, as discussed elsewhere, since the a_i 's have no units of measurement, then they do not have to represent fractions of costs. As with the GBAS, any other value measure consistent with the firm's decision-making objective for the allocation itself could be substituted for indirect costs and still keep the various properties of the original schemes. The last example of Table 2 serves to illustrate this point. There, the units of measurement is the annual demand. The firm appears to encourage sales volume, by allocating an indirect cost amount, X_i , to centre i which is lower than that assigned by any other allocation scheme. The end result is a substantial generalization of the existing schemes. Allocating the coalition benefits in proportion to ability-to-bear measures ties the most widely used activity-based allocation basis to the best known normative models.

Appendix B

Proof of Property 3d. The following definitions are needed to prove Property 3d.

Definition B.1. A function $f(x): \mathbb{R}^n \rightarrow \mathbb{R}$ is said to be symmetric iff $f(x) = f(x^a)$, where x^a is one of the $n!$ permutations of x .

Example. If $x = (x_1, x_2) \in \mathbb{R}^2$, then $f(x): \mathbb{R}^n \rightarrow \mathbb{R} = (x_1, x_2)$ is a symmetric function.

Definition B.2. A set $S \in \mathbb{R}^n$ is said to be symmetric iff

$$x \in S \quad x^a \in S,$$

where x^a is one the $n!$ permutations of x .

Example. The unit circle in \mathbb{R}^2 with its centre at the origin is a symmetric set.

Using the above definitions, the following lemma justifies Property 3d. We believe that this lemma is part of the folklore, but for the sake of completeness, we state the proof here.

Lemma. Consider the following problem:

$$\begin{array}{ll} \text{Max (Min)} & F(X) \\ \text{subject to} & X \in S, \end{array}$$

where $X = (X_1, X_2, \dots, X_n)$, $X \in \mathbb{R}_n$.

If $F(X)$ is a concave (convex) symmetric function of X , and S is a compact, convex, symmetric set, \exists an optimal solution X^* to the problem where $X_1^* = X_2^* = \dots = X_n^*$.

Proof. Since $F(X)$ is a continuous function and S is compact, it follows that an optimal solution exists to the above problem — say it exists at $X^a = (X_1^a, X_2^a, \dots, X_n^a)$. Then, X_q^a , $1 \leq q \leq n!$, where X_q^a is one of the $n!$ permutations of X^a , is also in S (because S is symmetric) and

$$F(X_q^a) = F(X^a), \quad 1 \leq q \leq n!$$

(because F is symmetric). Consider the following point:

$$X^* = \sum_{q=1}^{n!} X_q^a / n!$$

$X^* \in S$ because S is convex and X^* is a convex combination of points in S . In addition,

$$F(X^*) \begin{cases} \geq \sum_{q=1}^{n!} F(X_q^a) / n! & \text{if } F \text{ is concave,} \\ \leq \sum_{q=1}^{n!} F(X_q^a) / n! & \text{if } F \text{ is convex,} \end{cases}$$

implying that $F(X^*) = F(X^a)$. Furthermore, $X_1^a = X_2^a = \dots = X_n^a$. ■

References:

- [1] Balachandran, B.V., and Ramakrishnan, R.T.S., "Joint cost allocation: A unified approach", *The Accounting Review*, 56 (1981) 85-96.
- [2] Cheng, C.S.A., and Liao, W.M., "Simultaneous determination of joint product cost allocations and cost-plus prices", *Decision Sciences* 23 (1992) 785-796.

- [3] Demski, J.S., and Magee, R.P., "A perspective on accounting for defense contracts", *The Accounting Review* 67 (1992) 732-740.
- [4] Fremgen, J., and Liao, S., "The allocation of corporate indirect cost", National Association of Accountants, New York, 1981.
- [5] Gangolly, J.S., "On joint cost allocations: Independent cost proportional scheme (ICPS) and its properties", *Journal of Accounting Research* 19 (1981) 299-312.
- [6] Gerchak, Y., and Gupta, D., "On apportioning costs to customers in centralized continuous review systems", *Journal of Operations Management* 10 (1991) 546-551.
- [7] Hamlen, S., Hamlen, W., and Tschirhart, J., "The use of core theory in evaluating joint cost allocation schemes", *The Accounting Review* 52 (1977) 616-627.
- [8] Hamlen, S., Hamlen, W., and Tschirhart, J., "The use of the generalized shapley allocation in joint cost allocation", *The Accounting Review* 55 (1980) 269-287.
- [9] IE News, "Government cost probe may scuttle MRP II", *Production and Inventory Control* 22 (1987) 1-7.
- [10] Lichtenberg, F.R., "A perspective on accounting for defense contracts", *The Accounting Review* 67 (1992) 741-752.
- [11] Louderback, J.G., "Another approach to allocating joint costs: A comment", *The Accounting Review* 51 (1976) 683-685.
- [12] Moriarity, S., "Another approach to allocating joint costs", *The Accounting Review* 50 (1975) 791-795.
- [13] Murthy, K.G., *Linear Programming*, Wiley, New York, 1983.
- [14] Reichelstein, S., "Constructing incentive schemes for government contracts: An application of agency theory", *The Accounting Review* 67 (1992) 712-731.
- [15] Robinson, L.W., "A comment on Gerchak and Gupta's 'On apportioning costs to customers in centralized continuous review systems'", *Journal of Operations Management* 11 (1993) 99-102.
- [16] Rogerson, W.P., "Overhead allocation and incentives for cost minimization in defense procurement", *The Accounting Review* 67 (1992) 671-690.
- [17] Roth, A., and Verrecchia, R., "The Shapley value as applied to cost allocation: A reinterpretation", *Journal of Accounting Research* 17 (1979) 295-302.
- [18] Schneider, A., "Simultaneous determination of cost allocations and cost-plus prices for joint products", *Journal of Business Finance and Accounting* 13 (1986) 187-195.
- [19] Schneider, A., "Indirect cost allocation and cost-plus pricing formulas", *Journal of Cost Analysis* 3 (1986) 47-57.
- [20] Schneider, A., and Jeroslow, R.G., "Joint product cost allocation in the context of cost-plus pricing determinations with non-uniform markups", *Decision Sciences* 19 (1988) 424-433.
- [21] Shapley, L., "A value for n-person game", in: H.W. Kuhn and A.W. Tucker (eds.), *Contributions to the Theory of Games II*, Princeton University Press, Princeton, NJ, 1953.
- [22] Tersine, R.J., *Principles of Inventory and Materials Management*, North-Holland, Amsterdam, 1988.
- [23] Thomas, A.L., *The Allocation Problem, Part Two*, Studies in Accounting Research No. 9, American Accounting Association, 1974.
- [24] Thomas, J.K., and Tung, S., "Cost manipulation incentives under cost reimbursement: Pension costs for defense contracts", *The Accounting Review* 67 (1992) 691-711.
- [26] Zinn, W., Levy, M., and Bowersox, D.J., "Measuring the effect of inventory centralization/decentralization on aggregate stock: The 'Square Root Law' revisited" *Journal of Business Logistics* 10 (1989) 1-13.